Persistent Cohomology and Circle-valued coordinates

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January 19, 2009

Outline

Finding coordinates

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  Different coordinate choice might concentrate intrinsic information.
- Want: find few and very relevant intrinsic coordinates.

Problematic cases

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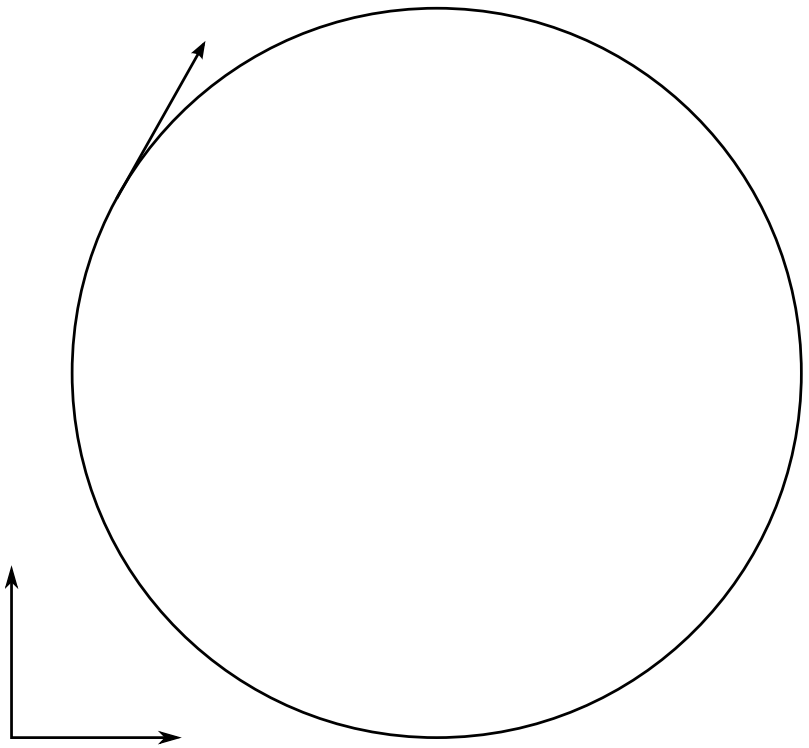

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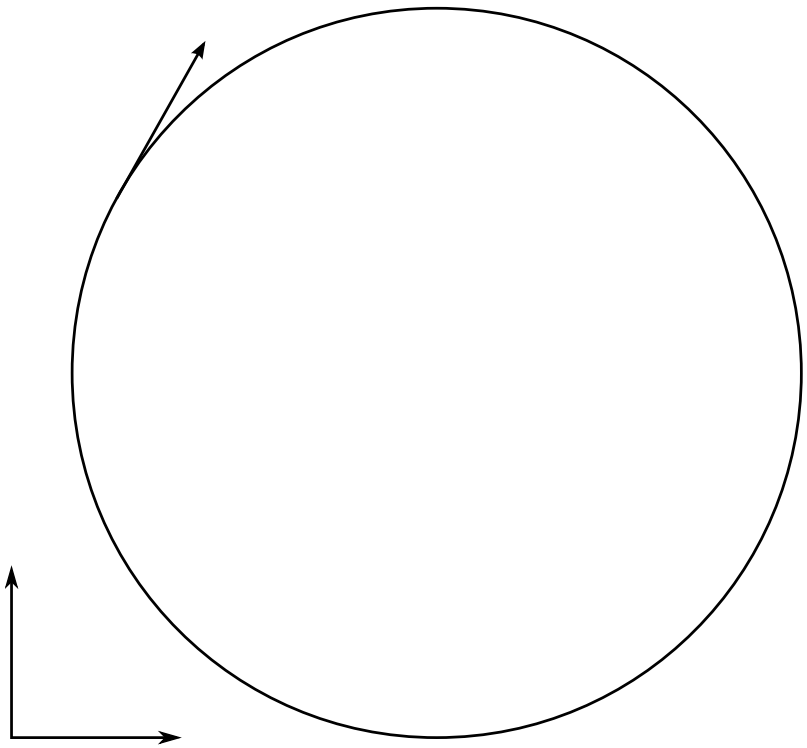

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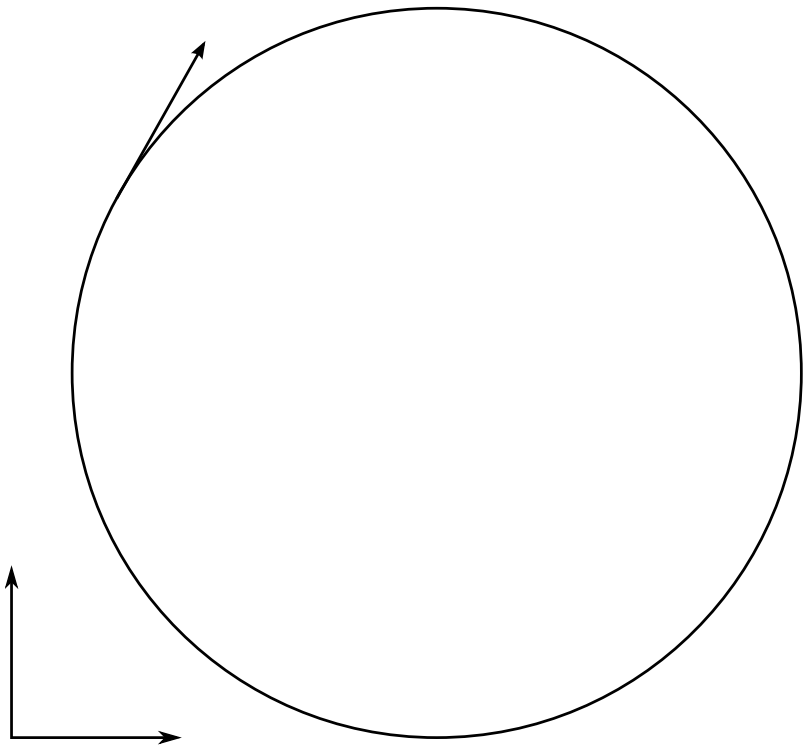

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Similar problems arise with sphere and torus.

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Circle-valued coordinates

- Use $S^1 = [0,1]/(0 \sim 1)$ as coordinate space
- Fixes the circle
- Fixes the torus
- Occurs naturally:
  - Phase coordinates for waves
  - Angle coordinates for directions

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Persistent Cohomology

- Degree one cohomology equivalent to circle-valued maps
- Persistence picks out relevant features
- Once a feature-rich parameter has been found, we work in ordinary (non-persistent) cohomology theories

From cohomology to circle-valued parametrizations

Use canonical isomorphism

$$H^1(X) \cong [X, S^1]$$

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  Smoothness: Integral cohomology gives constant values on all vertices, and wraps edges in the complex around the target circle.
- Numerical stability of cohomology computation and of the smoothing operations.

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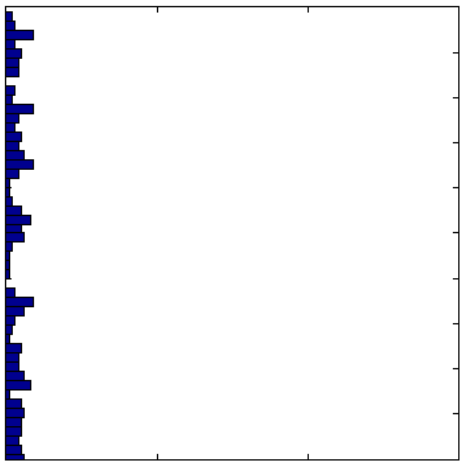
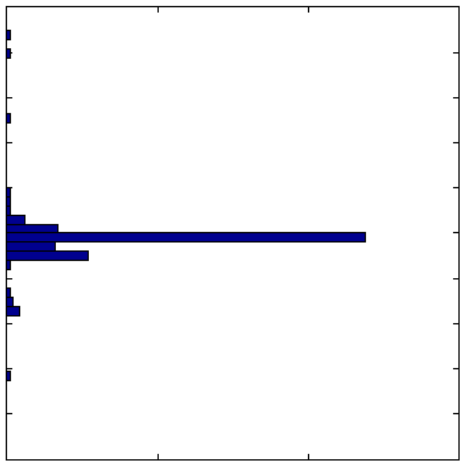
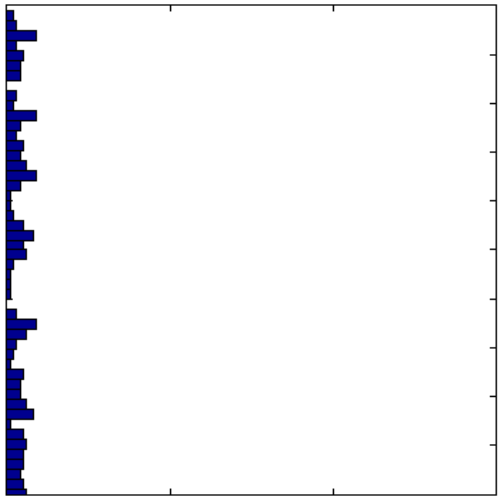
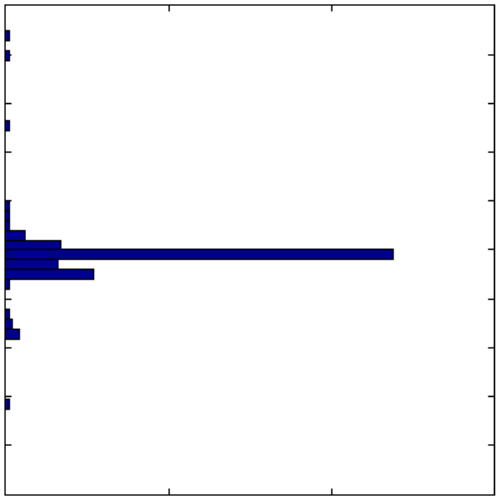
- The function might be constant almost everywhere
- Can find relevance by using a parameter value histogram

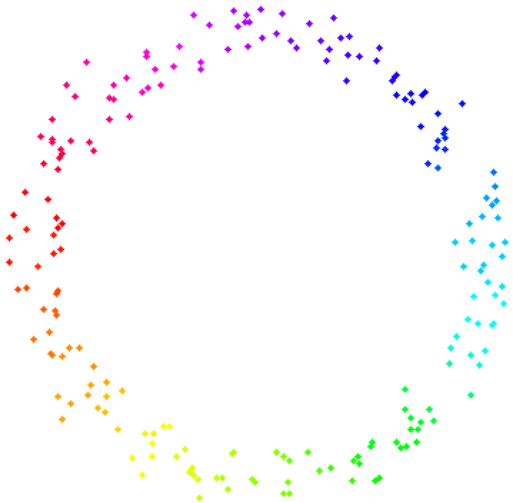


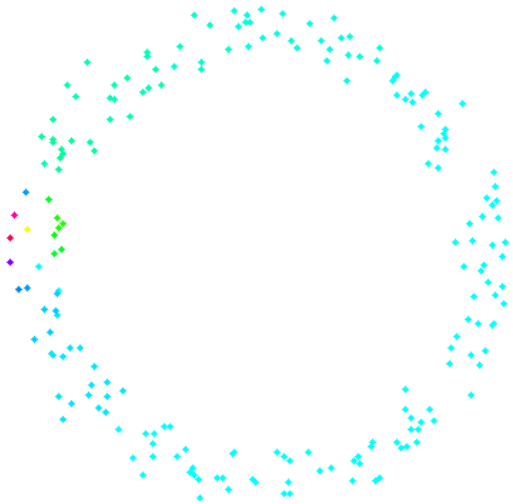


Interpreting histograms









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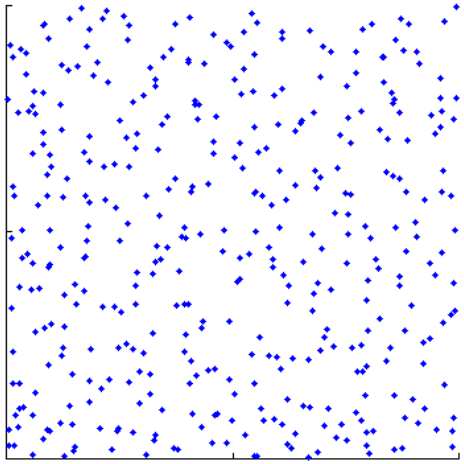

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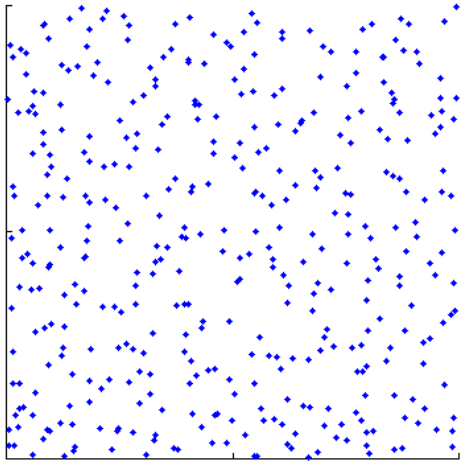

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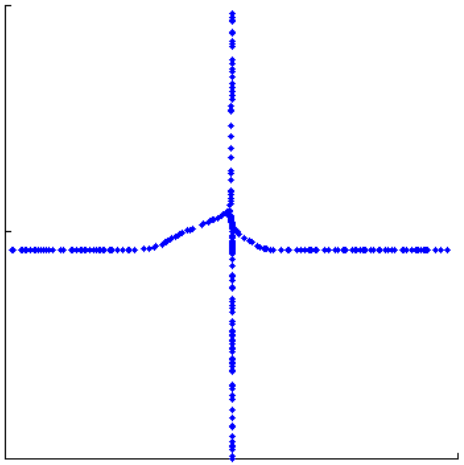

Correlation plot for a pair of circles

Time for a demo?

The end.

Any questions?